Summary: Let \( d \) and \( n \) be integers satisfying \( C \leq d \leq \exp(c \sqrt{\ln n}) \) for some universal constants \( c, C > 0 \), and let \( z \in \mathbb{C} \). Denote by \( M \) the adjacency matrix of a random \( d \)-regular directed graph on \( n \) vertices. In this paper, we study the structure of the kernel of submatrices of \( M - z, \text{Id} \), formed by removing a subset of rows. We show that with large probability the kernel consists of two nonintersecting types of vectors, which we call very steep and gradual with many levels. As a corollary, we show, in particular, that every eigenvector of \( M \), except for constant multiples of \((1, 1, \ldots, 1)\), possesses a weak delocalization property: its level sets have cardinality less than \( Cn \ln^2 d/\ln n \). For a large constant \( d \), this provides principally new structural information on eigenvectors, implying that the number of their level sets grows to infinity with \( n \). As a key technical ingredient of our proofs, we introduce a decomposition of \( Cn \) into vectors of different degrees of “structuredness”, which is an alternative to the decomposition based on the least common denominator in the regime when the underlying random matrix is very sparse.

MSC:
- 05C20 Directed graphs (digraphs), tournaments
- 05C80 Random graphs (graph-theoretic aspects)
- 60B20 Random matrices (probabilistic aspects)
- 15B52 Random matrices (algebraic aspects)
- 46B06 Asymptotic theory of Banach spaces
- 46B09 Probabilistic methods in Banach space theory
- 60C05 Combinatorial probability

Keywords:
delocalization of eigenvectors; Littlewood-Offord theory; random graphs; random matrices; regular graphs; sparse matrices; structure of kernel

Full Text: DOI arXiv

References:
[12] Bourgade, Paul; Huang, Jiaoyang; Yau, Horng-Tzer, Eigenvector statistics of sparse random matrices, Electron. J. Probab.,