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Stability results on the circumference of a graph. (English) [Zbl 1449.05163]

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Authors’ abstract: In this paper, we extend and refine previous Turán-type results on graphs with a given circumference. Let $W_{n,k,c}$ be the graph obtained from a clique $K_{c-k+1}$ by adding $n-(c-k+1)$ isolated vertices each joined to the same $k$ vertices of the clique, and let $f(n,k,c) = e(W_{n,k,c})$. Improving a celebrated theorem of P. Erdős and T. Gallai [Acta Math. Acad. Sci. Hung. 10, 337–356 (1959; Zbl 0090.39401)], G. N. Kopylov [Sov. Math., Dokl. 18, 593–596 (1977; Zbl 0419.05032); translation from Dokl. Akad. Nauk SSSR 234, 19–21 (1977)] proved that for $c < n$, any 2-connected graph $G$ on $n$ vertices with circumference $c$ has at most $\max\{f(n,2,c), f(n,\lfloor c/2 \rfloor, c)\}$ edges, with equality if and only if $G$ is isomorphic to $W_{n,2,c}$ or $W_{n,\lfloor c/2 \rfloor, c}$. Recently, Z. Füredi et al. [J. Comb. Theory, Ser. B 121, 197–228 (2016; Zbl 1348.05105)] proved a stability version of Kopylov’s theorem. Their main result states that if $G$ is a 2-connected graph on $n$ vertices with circumference $c$ such that $10 \leq c < n$ and $e(G) > \max\{f(n,k+1,c), f(n,\lfloor c/2 \rfloor + 1,c)\}$ then either $G$ is a subgraph of $W_{n,2,c}$ or $W_{n,\lfloor c/2 \rfloor, c}$ or $c$ is odd and $G$ is a subgraph of a member of two well-characterized families which we define as $X_{n,c}$ and $Y_{n,c}$. We prove that if $G$ is a 2-connected graph on $n$ vertices with minimum degree at least $k$ and circumference $c$ such that $10 \leq c < n$ and $e(G) > \max\{f(n,k+1,c), f(n,\lfloor c/2 \rfloor + 1,c)\}$, then one of the following holds:

(i) $G$ is a subgraph of $W_{n,k,c}$ or $W_{n,\lfloor c/2 \rfloor, c}$,
(ii) $k = 2$, $c$ is odd, and $G$ is a subgraph of a member of $X_{n,c} \cup Y_{n,c}$, or
(iii) $k \geq 3$ and $G$ is a subgraph of the union of a clique $K_{c-k+1}$ and some cliques $K_{k+1}$’s, where any two cliques share the same two vertices.

This provides a unified generalization of the above result of Füredi et al. [loc. cit.] as well as a recent result of B. Li and B. Ning [Linear Multilinear Algebra 64, No. 11, 2252–2269 (2016; Zbl 1352.05105)] and independently, of Z. Füredi et al. [Discrete Math. 340, No. 11, 2688–2690 (2017; Zbl 1369.05118)] on non-Hamiltonian graphs. A refinement and some variants of this result are also obtained. Moreover, we prove a stability result on a classical theorem of J. A. Bondy [ibid. 1, 121–132 (1971; Zbl 0224.05120)]. We use a novel approach, which combines several proof ideas including a closure operation and an edge-switching technique. We will also discuss some potential applications of this approach for future research.

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MSC:

05C45 Eulerian and Hamiltonian graphs
05C40 Connectivity
05C35 Extremal problems in graph theory
05C38 Paths and cycles
05D99 Extremal combinatorics

Keywords:

graph circumference; clique; closure operation; edge-switching technique; Hamiltonian graph; Woodall’s conjecture; 2-connected graph

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References:
