Kesler, R.; **Lacey, M. T.**

ℓ^p-improving inequalities for discrete spherical averages. (English) Zbl 1449.42033

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For $\lambda^2 \in \mathbb{N}$, let $\mathbb{S}^d_\lambda := \{n \in \mathbb{Z}^d \ : \ |n| = \lambda\}$. For a function $f : \mathbb{Z}^d \to \mathbb{R}$, define

$$A_\lambda f(x) = |\mathbb{S}^d_\lambda|^{-1} \sum_{n \in \mathbb{S}^d_\lambda} f(x-n).$$

The following estimate is the main result of the paper under review:

$$|A_\lambda|_{\ell^p \to \ell^{p'}} \leq C_{d,p,\omega(\lambda^2)} \lambda^{d(1-\frac{2}{p})}, \tfrac{d-1}{d+1} < p \leq \frac{d}{d-2}, d \geq 4.$$

In dimension $d = 4$ this estimate proved for odd λ^2. Here $\omega(\lambda^2)$ is the number of distinct prime factors of λ^2.

This inequality is a discrete version of a classical inequality of *W. Littman* [Partial diff. Equ., Berkeley 1971, Proc. Sympos. Pure Math. 23, 479–481 (1973; Zbl 0263.44006)] and *R. S. Strichartz* [J. Funct. Anal. 5, 218–235 (1970; Zbl 0189.40701)] on the L^p improving property of spherical averages on $\mathbb{R}^d$.

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