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A finite element variational multiscale method based on Crank-Nicolson scheme for the unsteady Navier-Stokes equations. (Chinese. English summary) [Zbl 1449.65264]

Summary: The incompressible viscous flows are fluid movements that do not change in density. They are used to describe many important physical phenomena such as weather, ocean currents, flow around airfoil, and blood flow within the arteries. The Navier-Stokes equations are the basic equations for incompressible viscous flows. Therefore, the numerical method for solving Navier-Stokes equations has been paid more and more attention in recent decades. In this paper, we mainly study a two-level fully discrete finite element variational multiscale method based on Crank-Nicolson scheme for the unsteady Navier-Stokes equations. The method is carried out in two steps. A stabilized nonlinear Navier-Stokes system is solved on a coarse grid at the first step, and the second step is that a stabilized linear problem is solved on a fine grid to correct the coarse grid solution. Error estimate of the velocity which is derived via the two-level finite element variational multiscale method is of second-order in time. Numerical experiments show that the method of this paper can save a lot of computation time compared with the finite element variational method which uses a one-level grid directly on the fine grid in the case of coarse grid matching.

MSC:
65M60 Finite element, Rayleigh-Ritz and Galerkin methods for initial value and initial-boundary value problems involving PDEs
65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N55 Multigrid methods; domain decomposition for boundary value problems involving PDEs
76D05 Navier-Stokes equations for incompressible viscous fluids
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
76M10 Finite element methods applied to problems in fluid mechanics
76M20 Finite difference methods applied to problems in fluid mechanics
35Q30 Navier-Stokes equations
65M15 Error bounds for initial value and initial-boundary value problems involving PDEs

Keywords:
Navier-Stokes equations; two-grid method; Crank-Nicolson scheme; error estimate

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