Nakamura, Yusuke; Tanaka, Hiromu
A Witt Nadel vanishing theorem for threefolds. (English) Zbl 1451.14124

The authors of the paper under review work over a perfect field $k$ of characteristic $p > 5$. A log-pair $(X, \Delta)$, where $X$ is normal and $\Delta$ is an $\mathbb{R}$-divisor, has klt singularities if for any birational morphism $\pi: Y \to X$ with $Y$ normal, one has $K_Y \sim \pi^*(K_X + \Delta) + \sum_i a_i E_i$, with $a_i > -1$ for all $i$. The non-klt locus of a log-pair $(X, \Delta)$ is the (reduced) subscheme $N_{\text{klt}}(X, \Delta)$ consisting of not-klt points. For any $k$-scheme $X$, let $I \subseteq O_X$ be a coherent ideal sheaf on $X$; one defines the associated sheaf $WI$ of Witt vectors as $WI(U) := W(I(U))$.

The main result of the paper under review, Theorem 4.10, is a characteristic-$p$ version of a vanishing theorem, in the spirit of the work by H. Esnault in [Invent. Math. 151, No. 1, 187–191 (2003; Zbl 1092.14010)]: more precisely, let $(X, \Delta)$ be a three-dimensional log-pair, $f: X \to Z$ a projective morphism to a quasi-projective scheme and suppose that $-(K_X + \Delta)$ is $f$-nef and $f$-big, then $R^i f_*(WI_{N_{\text{klt}}(X, \Delta), \mathbb{Q}}) = 0$ holds for all $i > 0$, where $I_{N_{\text{klt}}(X, \Delta), \mathbb{Q}}$ is the coherent ideal sheaf corresponding to $N_{\text{klt}}(X, \Delta)$. The strategy to prove this result consists in generalising the techniques by Y. Gongyo and the authors of the paper under review in [J. Eur. Math. Soc. (JEMS) 21, No. 12, 3759–3795 (2019; Zbl 1462.14045)] to reduce the statement to a $W\mathcal{O}$-vanishing for log Fano contractions (cf. Theorem 3.11).

As a consequence of the main result of the paper under review, the authors obtain several other results, such as the Kollár-Shokurov connectedness theorem (cf. Theorem 4.12), which generalises the work by C. Birkar in [Ann. Sci. Éc. Norm. Supér. (4) 49, No. 1, 169–212 (2016; Zbl 1346.14040)]. Another consequence is Corollary 5.2: suppose that $k$ is finite, then for a non-klt projective normal Fano threefold the number of rational points on the klt-locus is divisible by the cardinality of $k$.

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MSC:

14J45 Fano varieties
14E30 Minimal model program (Mori theory, extremal rays)
14F17 Vanishing theorems in algebraic geometry

Keywords:
rational points; Witt vectors; Nadel vanishing theorem

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