Tabachnikov, Serge
Projective configuration theorems: old wine into new wineskins. (English) Zbl 1451.51001

In this survey paper, the author considers modern results in the spirit of classical configuration theorems. First, the author recalls the classical projective configuration results of Pappus, Desargues, Pascal, Brianchon, and Poncelet. He then gives a modern general definition of a configuration theorem as one where given a collection of points and lines in a projective plane, when certain lines are constructed through pairs of points, or pairs of lines intersect at a point then certain incidence relations hold. This viewpoint allows the author to project and normalize old results in ways that would not necessarily have been natural to classical geometers, but which in turn encourages generalization and abstraction of sets of relations and exploration of their relationships with modern machinery.

The author does not attempt a comprehensive survey, choosing to select a half-dozen topics for which he sketches the main modern results, often driven by computer exploration. The topics the author explores are first the iterated Pappus theorem and its connection to the modular group, noting that determining the fractal dimension of Pappus curves in general is an open problem. Another area that derives from the Pappus theorem is the Steiner theorem and the twisted cubic.

The pentagram map is a transformation of the moduli space of projective equivalence classes of polygons in the projective plane and forms a discrete completely integral system. The third topic is a collection of eight configuration theorem of projective geometry dealing with pentagram-like maps on inscribed polygons. Some of these were discovered in an REU (Research Experiences for Undergraduates) summer program.

A Poncelet polygon is a polygon inscribed in an ellipse and circumscribed about another ellipse. A crucial role is played by the lines containing the edges of the polygon and their points of tangency with the ellipse. The author derives a number of results about the configurations of resulting points and lines using the billiard properties of conics.

The altitudes of Euclidean, spherical and hyperbolic triangles are concurrent. In the case of spaces of non-zero constant curvature, whether positive or negative, these results have an interpretation in terms of the Jacobi identities of the associated Lie algebras, $so(3)$ and $sl(2, \mathbb{R})$.

For the last topic, the author moves from the plane into 3-dimensional space and introduces the notion of a skewer of two lines, their common perpendicular. The author gives skewer analogues of the Pappus and Desargues configuration theorems and also considers the situation in both elliptic space and hyperbolic space.

The author covers a lot of territory quickly. No more definitions and technicalities are introduced than are needed to state the main results. Few proofs are given (except in the case of some unpublished results), but there are extensive references to the literature. The reader is given a rapid introduction to a number of active areas of research that hearken back to classical results, albeit in very modern dress.

For the entire collection see [Zbl 1426.01005].

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MSC:

- 51-03 History of geometry
- 01A05 General histories, source books
- 51A20 Configuration theorems in linear incidence geometry
- 54-03 History of general topology

Keywords:

- theorems of Pappus; Desargues; Pascal; Brianchon; Poncelet; Sylvester problem; billiards; pentagram map; skewers; line congruences
Software:

Cinderella

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References:


