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Summary: For a positive integer \( k \), let \( [k] = \{1, 2, \ldots, k\} \) and let \( \mathcal{P}([k]) \) denote the power set of the set \([k]\) and \( \mathcal{P}^t([k]) = \mathcal{P}([k]) - \emptyset \). For each integer \( t \) with \( 1 \leq t < k \), let \( \mathcal{P}_t([k]) \) denote the set of \( t \)-element subsets of \( \mathcal{P}([k]) \). For an edge coloring \( c : E(G) \to \mathcal{P}([k]) \) of a graph \( G \), where adjacent edges may be colored the same, \( c' : V(G) \to \mathcal{P}^t([k]) \) is the vertex coloring in which \( c'(v) \) is the union of the color sets of the edges incident with \( v \). If \( c' \) is a proper vertex coloring of \( G \), then \( c \) is a majestic \( t \)-tone \( k \)-coloring of \( G \). For a fixed positive integer \( t \), the minimum positive integer \( k \) for which a graph \( G \) has a majestic \( t \)-tone \( k \)-coloring is the majestic \( t \)-tone index \( \text{maj}_t(G) \) of \( G \). It is known that if \( G \) is a connected bipartite graph of order at least 3, then \( \text{maj}_t(G) = t + 1 \) or \( \text{maj}_t(G) = t + 2 \) for each positive integer \( t \). It is shown that (i) if \( G \) is a 2-connected bipartite graph of arbitrarily large order \( n \) whose longest cycles have length \( \ell \), where \( n - 5 \leq \ell \leq n \) and \( t \geq 2 \) is an integer, then \( \text{maj}_t(G) = t + 1 \) and (ii) there is a 2-connected bipartite graph \( F \) of arbitrarily large order \( n \) whose longest cycles have length \( n - 6 \) and \( \text{maj}_2(F) = 4 \). Furthermore, it is shown for integers \( k, t \geq 2 \) that there exists a \( k \)-connected bipartite graph \( G \) such that \( \text{maj}_t(G) = t + 2 \). Other results and open questions are also presented.

MSC:
05C15 Coloring of graphs and hypergraphs
05C38 Paths and cycles
05C75 Structural characterization of families of graphs

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majestic \( t \)-tone coloring; majestic \( t \)-tone index; bipartite graphs