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A combinatorial characterization of the Baer and the unital cone in $\mathrm{PG}(3, q^2)$**.** (English)

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The characterisation of point sets by their intersection numbers is a classical problem in finite geometry.

A set of points S in $\mathrm{PG}(n, q)$ is said to be of type $(m_1, m_2, \ldots, m_s)_d$ if the size of the intersection of any d-space with S is contained in $\{m_1, m_2, \ldots, m_s\}$ and all possibilities m_i occur as an intersection for some d-space.

It is a well-known result of *A. Bruen* [Bull. Am. Math. Soc. 76, 342–344 (1970; Zbl 0207.02601)] and *A. A. Bruen* and *J. A. Thas* [Geom. Dedicata 6, 193–203 (1977; Zbl 0367.05009)] that a set of type $(1, q+1)_1$ in $\mathrm{PG}(2, q^2)$ is either a Baer subplane or a unital.

In this paper, the authors deal with two particular examples of point sets with three intersection numbers with respect to planes. The first example is a Baer cone (a cone with vertex a point and base a Baer subplane), the second is a unital cone (a cone with vertex a point and base a unital). Both examples form 2-blocking sets in $\mathrm{PG}(3, q^2)$. A 2-blocking set in $\mathrm{PG}(3, q^2)$ is a blocking set with respect to lines.

The two main results of this paper show that a 2-blocking set of type $(q^2+1, q^2+q+1, q^3+q^2+1)_2$ is a Baer cone and a 2-blocking set of type $(q^2+1, q^3+1, q^3+q^2+1)_2$ is a unital cone.

The proofs are combinatorial. The hypothesis that the point sets form 2-blocking sets enables the authors to make use of the known results about planar blocking sets in $\mathrm{PG}(2, q^2)$.

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MSC:

51E20 Combinatorial structures in finite projective spaces
51E21 Blocking sets, ovals, k-arcs

Keywords:

three character sets; sets of type $(q^2+1, q^2+q+1, q^3+q^2+1)_2$; sets of type $(q^2+1, q^3+1, q^3+q^2+1)_2$; Baer cones; unital cones; blocking sets

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