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Empirical measures: regularity is a counter-curse to dimensionality. (English) Zbl 1452.60007

Summary: We propose a “decomposition method” to prove non-asymptotic bound for the convergence of empirical measures in various dual norms. The main point is to show that if one measures convergence in duality with sufficiently regular observables, the convergence is much faster than for, say, merely Lipschitz observables. Actually, assuming $s$ derivatives with $s > d/2$ ($d$ is the dimension) ensures an optimal rate of convergence of $1/\sqrt{n}$ ($n$ is the number of samples). The method is flexible enough to apply to Markov chains which satisfy a geometric contraction hypothesis, assuming neither stationarity nor reversibility, with the same convergence speed up to a power of logarithmic factor. Our results are stated as controls of the expected distance between the empirical measure and its limit, but we explain briefly how the classical method of bounded difference can be used to deduce concentration estimates.

MSC:

60B10 Convergence of probability measures
60J05 Discrete-time Markov processes on general state spaces
62E17 Approximations to statistical distributions (nonasymptotic)
49Q20 Variational problems in a geometric measure-theoretic setting

Keywords:

concentration; dual norms; empirical measure; Markov chains; non-asymptotic bounds; Wasserstein metric

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References:


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