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Summary: The eigenvalue problem of a graph Laplacian matrix $L$ arising from a simple, connected and undirected graph has been given more attention due to its extensive applications, such as spectral clustering, community detection, complex network, image processing and so on. The associated matrix $L$ is symmetric, positive semi-definite, and is usually large and sparse. It is often of interest for finding some smallest positive eigenvalues and corresponding eigenvectors.

However, the singularity of $L$ makes the classical eigensolvers inefficient since we need to factorize $L$ for the purpose of solving large and sparse linear systems exactly. The next difficulty is that it is usually prohibitive to factorize $L$ generated by real network problems from big data such as social media transactional databases, and sensor systems because there is in general not only local connections.

In this paper, we propose a trimming to cure the singularity of $L$ according to its special property: zero row/column sum. This remedy technique leads us to solve a positive definite linear system reduced in one dimension and then recover the result to get a suitable solution of the original system involved in an eigensolver. Besides, we apply a deflating approach to exclude the influence of converged eigenvalues.


Numerical experiments reveal that the integrated eigensolver outperforms the classical Arnoldi/Lanczos method for computing some smallest positive eigeninformation especially when the LU factorization is not available.

MSC:
65F15 Numerical computation of eigenvalues and eigenvectors of matrices
05C50 Graphs and linear algebra (matrices, eigenvalues, etc.)

Keywords:
graph Laplacian matrix; eigenvalue problem; trimming; deflation; shift-invert residual Arnoldi; inexact eigensolver

Software:
JDQZ; KONECT; LAPACK; JDQR; PETSc; SLEPc; ARPACK

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References:
[32] Xu, Z., Cooperative control of dynamical systems: applications to autonomous vehicles, (2009), Springer-Verlag · Zbl 1171.93005
[34] P.M. Vaidya, Solving linear equations with symmetric diagonally dominant matrices by constructing good preconditioners, Tech. rep., Department of Computer Science, University of Maryland at College Park, 2007.
[37] Koutis, I.; Miller, G. L.; Peng, R., A nearly-m log n time solver for SDD linear systems, (IEEE 52nd Annual Symposium on


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