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The high-dimensional cohomology of the moduli space of curves with level structures. (English) Zbl 1453.14083

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Summary: We prove that the moduli space of curves with level structures has an enormous amount of rational cohomology in its cohomological dimension. As an application, we prove that the coherent cohomological dimension of the moduli space of curves is at least $g - 2$. Well known conjectures of Looijenga would imply that this is sharp.

MSC:

14H10	Families, moduli of curves (algebraic)
32G15	Moduli of Riemann surfaces, Teichmüller theory (complex-analytic aspects in several variables)
57K20	2-dimensional topology (including mapping class groups of surfaces, Teichmüller theory, curve complexes, etc.)
57S05	Topological properties of groups of homeomorphisms or diffeomorphisms

Keywords:

moduli space of curves; mapping class group

Software:

MathOverflow

Full Text: DOI arXiv

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