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**Curve counting on elliptic Calabi-Yau threefolds via derived categories.** (English)

Zbl 1453.14139

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Summary: We prove the elliptic transformation law of Jacobi forms for the generating series of Pandharipande-Thomas invariants of an elliptic Calabi-Yau threefold over a reduced class in the base. This proves part of a conjecture by Huang, Katz, and Klemm [*M.-x. Huang et al.*, J. High Energy Phys. 2015, No. 10, Paper No. 125, 80 p. (2015; Zbl 1388.81219)]. For the proof we construct an involution of the derived category and use wall-crossing methods. We express the generating series of PT invariants in terms of low genus Gromov-Witten invariants and universal Jacobi forms.

As applications we prove new formulas and recover several known formulas for the PT invariants of  $K3 \times E$ , abelian 3-folds, and the STU-model. We prove that the generating series of curve counting invariants for  $K3 \times E$  with respect to a primitive class on the  $K3$  is a quasi-Jacobi form of weight  $-10$ . This provides strong evidence for the Igusa cusp form conjecture.

**MSC:**

- 14N35 Gromov-Witten invariants, quantum cohomology, Gopakumar-Vafa invariants, Donaldson-Thomas invariants (algebraic aspects)
- 14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
- 14J32 Calabi-Yau manifolds (algebraic aspects)
- 14J30 3-folds

Cited in 5 Documents

**Keywords:**

Pandharipande-Thomas invariants; elliptic fibrations; Jacobi forms

**Full Text:** DOI arXiv

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