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Undirected zero-divisor graphs and unique product monoid rings. (English) Zbl 1453.16040

Let $R$ be an associative ring with identity and $Z(R)$ be its set of non-zero zero-divisors. The zero-divisor graph $\Gamma(R)$ of $R$, is the simple undirected graph with vertex set $Z^*(R) = Z(R) \setminus \{0\}$ where two distinct vertices $x$ and $y$ are adjacent if and only if $xy = 0$ or $yx = 0$. In fact, this is a generalization of the zero divisor graph of a commutative ring $R$ introduced by D. F. Anderson and P. S. Livingston [J. Algebra 217, No. 2, 434–447 (1999; Zbl 0941.05062)] and studied by several authors for the past three decades. The distance between vertices $a$ and $b$ is the length of the shortest path connecting them, and the diameter of the graph, $\text{diam}(\Gamma(R))$, is the supremum of these distances between all pairs of distinct vertices in $\Gamma(R)$.

In this paper, the authors first prove some results about $\Gamma(R)$, where $R$ is a semi-commutative ring or a reversible ring or a unique product monoid $M$. Using those results, it is proved that $0 \leq \text{diam}(\Gamma(R)) \leq \text{diam}(\Gamma(R[M])) \leq 3$. Having obtained upper and lower bounds for the diameter of $\Gamma(R)$ and $\Gamma(R[M])$, they characterize all the possibilities for the pair $\text{diam}(\Gamma(R))$ and $\text{diam}(\Gamma(R[M]))$, strictly in terms of the properties of the underlying ring $R$, where $R$ is a reversible ring and $M$ is a unique product monoid. At the last part of the paper, the authors provide an example showing the necessity of the assumptions used in the statement of the result.

Reviewer: T. Tamizh Chelvam (Tirunelveli)

MSC:
16U99 Conditions on elements
13A99 General commutative ring theory
16S15 Finite generation, finite presentability, normal forms (diamond lemma, term-rewriting)
05C12 Distance in graphs

Keywords:
zero-divisor graph; diameter; semi-commutative ring; unique product monoid; monoid ring

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