Nop, G. N.; Romanowska, A. B.; Smith, J. D. H.

Category theory as a foundation for the concept analysis of complex systems and time series. (English) Zbl 1454.06003

Kuś, Marek (ed.) et al., Category theory in physics, mathematics, and philosophy. Proceedings of the conference "Category Theory in Physics, Mathematics and Philosophy", Warsaw, Poland, November 16–17, 2017. Cham: Springer; Warsaw: International Center for Formal Ontology. Springer Proc. Phys. 235, 119-134 (2019).

Summary: Wille's formal concept analysis, Hardegree's treatment of natural kinds, and Birkhoff's mathematical theory of polarities provide essentially equivalent tools for the analysis of a static system functioning at a single level. We now show how a number of categorical notions allow these tools to be extended to cover the analysis of complex systems involving multiple hierarchical levels indexed by a semilattice, including the case where a chain represents a time series governing the evolution of a single system. A semilattice is a poset category with finite products or coproducts. Our analysis then rests on functors from a semilattice to the category of complete lattice homomorphisms, or from a semilattice to a category of polarities and bonding relations.

For the entire collection see [Zbl 1429.18001].

MSC:

06B23 Complete lattices, completions
06B75 Generalizations of lattices
06A15 Galois correspondences, closure operators (in relation to ordered sets)
18B35 Preorders, orders, domains and lattices (viewed as categories)

Full Text: DOI

References:

[1] G. Birkhoff, Lattice Theory, 3rd edn. (American Mathematical Society, Providence, 1967) · Zbl 0153.02501

[2] B. Ganter, R. Wille, Formale Begriffsanalyse (Springer, Berlin, 1996) Translated by C. Franzke as Formal Concept Analysis (Springer, Berlin, 1999)

[3] G. Gierz, A. Romanowska, Duality for distributive bisemilattices. J. Aust. Math. Soc. 51, 247-275 (1991) · Zbl 0751.06008 · doi:10.1017/S1446788700034224

[4] G.M. Hardegree, An approach to the logic of natural kinds. Pac. Phil. Q. 63, 122-132 (1982)· doi:10.1111/j.1468-0114.1982.tb00093.x

[5] R. Hartshorne, Algebraic Geometry (Springer, Berlin, 1977) · Zbl 0367.14001 · doi:10.1007/978-1-4757-3849-0

[6] P.T. Johnstone, Stone Spaces (Cambridge University Press, Cambridge, 1982) · Zbl 0499.54001

[7] H.M. MacNeille, Partially ordered sets. Trans. Am. Math. Soc. 42, 416-460 (1937) · Zbl 63.0833.04 · doi:10.1090/S0002-9947-1937-1501929-X

[8] R. Padmanabhan, Regular identities in lattices. Trans. Am. Math. Soc. 158, 179-188 (1971) · Zbl 0249.06005 · doi:10.1090/S0002-9947-1971-0281661-3

[9] J. Płonka, On distributive quasilattices. Fundam. Math. 60, 191-200 (1967) · Zbl 0154.00709 · doi:10.4064/fm-60-2-191-200

[10] J. Płonka, On a method of construction of abstract algebras. Fundam. Math. 61, 183-189 (1967) · Zbl 0168.26701 · doi:10.4064/fm-61-2-183-189

[11] A.B. Romanowska, J.D.H. Smith, Modes (World Scientific, River Edge, 2002) · Zbl 1012.08001 · doi:10.1142/4953

[12] A.B. Romanowska, J.D.H. Smith, Duality for quasilattices and Galois connections. Fund. Inf. 156, 331-359 (2017) · Zbl 1425.06003

[13] J.D.H. Smith, A.B. Romanowska, Post-Modern Algebra (Wiley, New York, 1999) · Zbl 0946.00001 · doi:10.1002/9781118032589

[14] R.