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A proof of the multijoints conjecture and Carbery's generalization. (English) Zbl 1454.14130

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The Joints Theorem provides an upper bound for the number of intersection points of d lines with independent directions from a given set L of lines in the vector space $\mathbb{F}^d$ over a field $\mathbb{F}$. The intersection points of d lines in this sense are called a *joint*. In the Multijoints Theorem, the set L is replaced by d families of lines $L_1, \dots, L_d$ each of which is required to contain precisely one of the intersecting lines.

In this article, the author proves, for the first time, the Multijoint Theorem without restriction on dimension d or field $\mathbb{F}$ and without genericity assumptions on the joints. Moreover, he proves Carbery's conjecture which provides a bound on the sum over all joint multiplicities and can be seen as a discrete version of the multilinear Kakeya theorem.

A key concept in the proof is a "generalized intersection multiplicity" of a line and a polynomial hypersurface at an intersection point that behaves reasonably well, even if the line is contained in the hypersurface, and allows a Bézout type upper bound. This simplifies application of the so called "polynomial method" in proofs as the distinction between lines on and off the polynomial hypersurface becomes less relevant.

Reviewer: Hans-Peter Schröcker (Innsbruck)

MSC:

14N05 Projective techniques in algebraic geometry
51E30 Other finite incidence structures (geometric aspects)

Keywords:

joint; multijoint; polynomial method

Full Text: DOI arXiv

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