Define the induced unitary operator by $U_T : L^2_m(X) \to L^2_m(X)$, $(U_T f)(x) := f(Tx)$. Let $T$ be a measure-preserving transformation on some measure space. The automorphism $T$ is weak mixing if and only if

$$\lim_{L \to \infty} \frac{1}{L^\alpha} \sum_{k=0}^{L-1} |\langle f, U^k_T f \rangle|^2 = 0$$

for all $f \in \mathbb{1}^\perp$, where $f \in \mathbb{1}^\perp$ is the orthogonal complement of the constant functions in $L^2_m(X)$ and $(f, g) = \int_X f g \, dm$ denotes the inner product in this space.

Let $G$ be the complete topological group of automorphisms of a Lebesgue space $(X, \mathcal{A}, m)$. In this paper, for each $\alpha \in [0, 1]$, each $f \in \mathbb{1}^\perp$ and each $T \in G$, the authors introduce the following:

$$M_{\alpha}(T; f) := \liminf_{L \to \infty} \frac{1}{L^\alpha} \sum_{k=0}^{L-1} |\langle f, U^k_T f \rangle|^2$$

and

$$M^\alpha(T; f) := \limsup_{L \to \infty} \frac{1}{L^{1-\alpha}} \sum_{k=0}^{L-1} |\langle f, U^k_T f \rangle|^2.$$

The following statement is proved.

**Theorem.** There exists a generic set $\mathcal{G} \subseteq G$ of weak-mixing automorphisms such that, for each $T \in \mathcal{G}$, the set

$$\{ f \in \mathbb{1}^\perp | M_{\alpha}(T; f) = 0 \text{ and } M^\alpha(T; f) = \infty, \forall 0 < \alpha < 1 \}$$

is generic in $\mathbb{1}^\perp$.

Then, the authors present generic sets of measures with respect to topological shifts over finite alphabets and Axiom A diffeomorphisms over topologically mixing basic sets.

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**MSC:**

37A05 Dynamical aspects of measure-preserving transformations
37A10 Dynamical systems involving one-parameter continuous families of measure-preserving transformations
37A25 Ergodicity, mixing, rates of mixing
28D05 Measure-preserving transformations

**Keywords:**
scales of mixing; Koopman operator; Halmos lemma; Rohlin lemma

**Full Text:** DOI

**References:**


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