Define the induced unitary operator by $U_T : L^2_\mu(X) \to L^2_\mu(X)$, $(U_T f)(x) := f(Tx)$. Let $T$ be a measure-preserving transformation on some measure space. The automorphism $T$ is weak mixing if and only if
\[
\lim_{L \to \infty} \frac{1}{L} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2 = 0 \quad \text{for all } f \in [1]^{\perp},
\]
where $f \in [1]^{\perp}$ is the orthogonal complement of the constant functions in $L^2_\mu(X)$ and $(f, g) = \int_X fg \, dm$ denotes the inner product in this space.

Let $G$ be the complete topological group of automorphisms of a Lebesgue space $(X, \mathcal{A}, m)$. In this paper, for each $\alpha \in [0, 1]$, each $f \in [1]^{\perp}$ and each $T \in G$, the authors introduce the following:

\[
\mathcal{M}_\alpha(T; f) := \liminf_{L \to \infty} \frac{1}{L^\alpha} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2
\]

and

\[
\mathcal{M}^\alpha(T; f) := \limsup_{L \to \infty} \frac{1}{L^{1-\alpha}} \sum_{k=0}^{L-1} |(f, U^k_T f)|^2.
\]

The following statement is proved.

Theorem. There exists a generic set $\mathcal{G} \in G$ of weak-mixing automorphisms such that, for each $T \in \mathcal{G}$, the set

\[
\{ f \in [1]^{\perp} | \mathcal{M}_\alpha(T; f) = 0 \quad \text{and} \quad \mathcal{M}^\alpha(T; f) = \infty, \forall 0 < \alpha < 1 \}
\]

is generic in $[1]^{\perp}$.

Then, the authors present generic sets of measures with respect to topological shifts over finite alphabets and Axiom A diffeomorphisms over topologically mixing basic sets.

Reviewer: Hasan Akin (Gaziantep)

MSC:

37A05 Dynamical aspects of measure-preserving transformations
37A10 Dynamical systems involving one-parameter continuous families of measure-preserving transformations
37A25 Ergodicity, mixing, rates of mixing
28D05 Measure-preserving transformations

Keywords:
scales of mixing; Koopman operator; Halmos lemma; Rohlin lemma

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.