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Zhang, Lei

Abundance for 3-folds with non-trivial Albanese maps in positive characteristic. (English)
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In recent years, many of the results of the Minimal Model Program have been extended from characteristic zero to characteristic $p > 5$. For example, existence of log minimal models of threefolds has been proved by *C. Birkar* [Ann. Sci. Éc. Norm. Supér. (4) 49, No. 1, 169–212 (2016; Zbl 1346.14040)], *C. D. Hacon* and *C. Xu* [J. Am. Math. Soc. 28, No. 3, 711–744 (2015; Zbl 1326.14032)]. There has also been significant progress on the log abundance conjecture. In the case of threefold klt pairs, it has been proved when the variety is of log general type or when the boundary divisor is big (in addition to the above references, see also [*P. Cascini* et al., Ann. Sci. Éc. Norm. Supér. (4) 48, No. 5, 1239–1272 (2015; Zbl 1408.14020)] and [*C. Xu*, J. Inst. Math. Jussieu 14, No. 3, 577–588 (2015; Zbl 1346.14020)]).

In the present paper, the author builds upon his previous work [*L. Zhang*, J. Lond. Math. Soc., II. Ser. 99, No. 2, 332–348 (2019; Zbl 1410.14013)] and proves abundance for threefolds with non-trivial Albanese map.

Theorem 1.1. Let X be a klt, $\mathbb{Q}$-factorial, projective minimal threefold defined over an algebraically closed fields k of characteristic $p > 5$. Assume that the Albanese map is non-trivial. Then K_X is semi-ample.

The author also proves some instances of log abundance.

Theorem 1.2. Let (X, B) be a klt, $\mathbb{Q}$-factorial, projective minimal pair of dimension three defined over an algebraically closed field of characteristic $p > 5$. Assume that the Albanese map α_X is non-trivial. Denote by $f : X \to Y$ the fibration arising from the Stein factorization of α_X and by X_η the generic fiber of f. Assume moreover that $B = 0$ if

(1) $\dim(Y) = 2$ and $\kappa(X_\eta, (K_X + B)|_{X_\eta}) = 0$, or

(2) $\dim(Y) = 1$ and $\kappa(X_\eta, (K_X + B)|_{X_\eta}) = 1$.

Then $K_X + B$ is semi-ample.

Reviewer: Justin Lacini (Lawrence)

MSC:

14E30	Minimal model program (Mori theory, extremal rays)
14E05	Rational and birational maps

Keywords:

abundance; positive characteristic; Albanese map

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