Kozlov, V. V.

The Liouville equation as a Hamiltonian system. (English. Russian original) Zbl 1455.37054
Math. Notes 108, No. 3, 339-343 (2020); translation from Mat. Zametki 108, No. 3, 360-365 (2020).

The author considers smooth dynamical systems on closed manifolds with invariant measure. The evolution of the density of a nonstationary invariant measure is described by the well-known Liouville equation and for ergodic dynamical systems, the latter is expressed in Hamiltonian form. An infinite collection of quadratic invariants that are pairwise in involution with respect to the Poisson bracket generated by the Hamiltonian structure is indicated. Some remarks and open questions are mentioned at the end of the paper.

Reviewer: Ahmed Lesfari (El Jadida)

MSC:

37K10 Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)
37K06 General theory of infinite-dimensional Hamiltonian and Lagrangian systems, Hamiltonian and Lagrangian structures, symmetries, conservation laws

Keywords:

Liouville equation; Hamiltonian systems; integrability

Full Text: DOI

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