D’Angeli, Daniele; Rodaro, Emanuele; Wächter, Jan Philipp

Orbit expandability of automaton semigroups and groups. (English) Zbl 1455.68107

The article studies several problems related to orbital growth in automaton semigroups and groups, where automaton semigroups are understood to be generated by partial deterministic letter-to-letter transducers. Consider an automaton semigroup $S$ acting on $\Sigma^*$ and for each $w \in \Sigma^*$ denote by $Sw$ the orbit of $w$ with respect to $S$. A word $u \in \Sigma^*$ is said to be $k$-expandable with respect to $S$ if there exists $v \in \Sigma^+$ such that $|Swv| \geq |Su| + k$ and expandable if it is $k$-expandable for some $k \geq 1$. This notion is motivated by the study of $\omega$-words with infinite orbits, as all prefixes of these ones are clearly expandable.

In the main result of the article, the authors prove that, given a transducer $T$ generating an automaton semigroup $S$, a word $u \in \Sigma^*$ and a natural number $k$, it is decidable whether $u$ is $k$-expandable with respect to $S$. A space-bounded nondeterministic decision procedure is described for this problem, while the Immerman-Szelepcsényi inductive counting technique is employed in its analysis [N. Immerman, SIAM J. Comput. 17, No. 5, 935–938 (1988; Zbl 0668.68056); R. Szelepcsényi, Acta Inf. 26, No. 3, 279–284 (1988; Zbl 0638.68046)].

The results for automaton semigroups are further strengthened in the case of an automaton group $G$. The authors prove an algebraic characterisation of expandable words with respect to $G$, which they use to obtain a more efficient decision procedure for this particular case.

It is also proved that every word is expandable in an automaton semigroup generated by a complete reversible transducer.

Reviewer: Peter Kostolányi (Bratislava)

MSC:

68Q70 Algebraic theory of languages and automata
20F10 Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)
20M35 Semigroups in automata theory, linguistics, etc.

Keywords:

automaton semigroup; automaton group; orbital growth; expandable word; decidability

Full Text: DOI arXiv

References:

[2] D’Angeli, Daniele; Francoeur, Dominik; Rodaro, Emanuele; Wächter, Jan Philipp, Orbits of automaton semigroups and groups (2019), arXiv preprint


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