If $X$ is a non-degenerate, irreducible, embedded projective variety over an algebraically closed field $k$ corresponding to a homogeneous prime ideal $P \subseteq S = k[x_1, \ldots, x_n]$, the Eisenbud-Goto conjecture predicts an upper bound for the regularity of $X$: $\text{reg}(X) \leq \deg(X) - \text{codim}(X) + 1$. This was open for many years, with special cases proved. Thus it was a bit of a shock when Peeva and the second author gave counterexamples to this conjecture, producing irreducible projective varieties with regularity much larger than their degrees. The two main new ideas in their work were so-called Rees-like algebras and step-by-step homogenization. All of the varieties thus produced are singular, and the current paper studies the singularities and their geometry. The authors compute the codimension of the singular locus of a Rees-like algebra over a polynomial ring, and then show that the step-by-step process can decrease the codimension of this singular locus. Thus the authors introduce prime standardization, as an alternative to step-by-step homogenization that preserves the codimension of the singular locus. They then look at the regularity of certain smooth hyperplane sections of Rees-like algebras and show that they all satisfy the Eisenbud-Goto conjecture. They also give a characterization of Rees-like algebras of Cohen-Macaulay ideals, and more generally they characterize when Rees-like algebras are seminormal, weakly normal, and in the case of positive characteristic, F-split.

Reviewer: Juan C. Migliore (Notre Dame)

MSC:

13D02 Syzygies, resolutions, complexes and commutative rings
13A35 Characteristic $p$ methods (Frobenius endomorphism) and reduction to characteristic $p$; tight closure
13A30 Associated graded rings of ideals (Rees ring, form ring), analytic spread and related topics
14J17 Singularities of surfaces or higher-dimensional varieties
13D05 Homological dimension and commutative rings
13F20 Polynomial rings and ideals; rings of integer-valued polynomials

Keywords:
Rees-like algebra; regularity; seminormal; F-split; singular locus; prime standardization; step-by-step homogenization

Software:
Macaulay2; Seminormalization; GitHub

Full Text: DOI

References:


Ene, V.; Herzog, J., Gröbner bases in commutative algebra, graduate studies in mathematics (2012), Providence: American Mathematical Society, Providence - Zbl 1242.13001


Niu, W., Castelnuovo-Mumford regularity bounds for singular surfaces, Math. e Zeit, 280, 3-4 (2020)


Vasconcelos, W., Computational methods in commutative algebra and algebraic geometry, algorithms and computation in mathematics (1998), Berlin: Springer-Verlag, Berlin - doi:10.1007/978-3-642-58951-5


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.