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Representability theorem in derived analytic geometry. (English) Zbl 1456.14018

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In the paper under review, the authors prove a representability theorem in derived analytic geometry, analogous to Lurie's generalization of Artin's representablility criteria to derived algebraic geometry. This is an important, standard type result for the study of moduli problems and a crucial step towards a solid theory of derived analytic geometry. More specifically, the authors show that a derived stack for the étale site of derived analytic spaces is a derived analytic stack if and only if it is compatible with Postnikov towers, has a global analytic cotangent complex, and its truncation is an analytic stack in the classical (underived) sense. The result applies both to complex analytic geometry and non-archimedean analytic geometry.

Central to representability results as in the present paper is deformation theory, which the authors develop here for the derived analytic setup. The authors define an analytic version of the cotangent complex which controls the deformation theory of the derived stack. As in the algebraic setting, the cotangent complex represents a functor of derivations. One key step in order to define the analytic cotangent complex is the elegant description of the ∞-category of modules over a derived analytic space X as the ∞-category of spectrum objects of a certain ∞-category associated with X. Another important construction is the analytification functor which they establish in the derived setting.

To apply derived geometry to classical moduli problems, one may try to enrich classical moduli spaces with derived structures. The paper under review is an important tool in verifying when such enrichments are indeed the correct ones.

Reviewer: Eric Ahlqvist (Stockholm)

MSC:

14D23	Stacks and moduli problems
14G22	Rigid analytic geometry
32G13	Complex-analytic moduli problems
14A30	Fundamental constructions in algebraic geometry involving higher and derived categories (homotopical algebraic geometry, derived algebraic geometry, etc.)

Cited in **1** Review
Cited in **3** Documents

Keywords:

representability; deformation theory; analytic cotangent complex; derived geometry; rigid analytic geometry; complex geometry; derived stacks

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