The Cremona group \( \text{Cr}_2(k) \) is the group of birational transformations of the projective plane \( \mathbb{P}^2 \) over a field \( k \). It was a long-standing question whether \( \text{Cr}_2(\mathbb{C}) \) is simple group. Several years ago S. Cantat et al. made a breakthrough in [Acta Math. 210, No. 1, 31–94 (2013; Zbl 1278.14017)] by proving that \( \text{Cr}_2(\mathbb{C}) \) is not simple. A. Lonjou generalized the result to an arbitrary field in [Ann. Inst. Fourier 66, No. 5, 2021–2046 (2016; Zbl 1365.14017)]. Over the field of complex numbers it was classically known that \( \text{Cr}_2(k) \) does not admit any non trivial homomorphism to an abelian group. Over the field of real numbers S. Zimmermann proved in [Duke Math. J. 167, No. 2, 211–267 (2018; Zbl 1402.14015)] that the abelianization of \( \text{Cr}_2(\mathbb{R}) \) is a direct sum of uncountably many \( \mathbb{Z}/2\mathbb{Z} \).

The article under review deals with an arbitrary perfect field with at least one Galois extension of degree eight. The authors constructed a tree on which \( \text{Cr}_2(k) \) acts so that \( \text{Cr}_2(k) \) can be written as an amalgam product by Bass-Serre theory. Note that each factor in the amalgam product is a big group and there are a lot of factors (same cardinality as the field \( k \)). Consequently the authors constructed a homomorphism from \( \text{Cr}_2(k) \) to a free product of \( \mathbb{Z}/2\mathbb{Z} \), thus also a homomorphism from \( \text{Cr}_2(k) \) to a direct sum of \( \mathbb{Z}/2\mathbb{Z} \).

The tree mentioned above comes from a square complex constructed in this paper on which \( \text{Cr}_2(k) \) acts. The vertices of the square are rank 1 fibrations with 1, 2, 3; rank 2 fibrations are generalizations of Mori fiber spaces. Roughly speaking the edges and the faces of the square complex record Sarkisov links and relations among Sarkisov links. If we blow up a general point of degree eight on \( \mathbb{P}^2 \) then we obtain a del Pezzo surface of degree 1. Such a del Pezzo surface gives a rank 2 fibration and an element in \( \text{Cr}_2(k) \) called a Bertini involution. This is where the hypothesis on the field \( k \) is used. Roughly speaking the tree is constructed by recording the action of \( \text{Cr}_2(k) \) on the part of the square complex containing these Bertini involutions.

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MSC:

14E07 Birational automorphisms, Cremona group and generalizations
14E30 Minimal model program (Mori theory, extremal rays)

Keywords:

Cremona group; Sarkisov program; amalgam product

Full Text: DOI

References:


Zimmermann, S.

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