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**Forward controllability of a random attractor for the non-autonomous stochastic sine-Gordon equation on an unbounded domain.** (English) [Zbl 1456.35043](#)

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Summary: A pullback random attractor is called forward controllable if its time-component is semi-continuous to a compact set in the future, and the minimum among all such compact limit-sets is called a forward controller. The existence of a forward controller closely relates to the forward compactness of the attractor, which is further argued by the forward-pullback asymptotic compactness of the system. The abstract results are applied to the non-autonomous stochastic sine-Gordon equation on an unbounded domain. The existence of a forward compact attractor is proved, which leads to the existence of a forward controller. The measurability of the attractor is proved by considering two different universes.

**MSC:**

[35B41](#) Attractors

[35L71](#) Second-order semilinear hyperbolic equations

[35L15](#) Initial value problems for second-order hyperbolic equations

[35R60](#) PDEs with randomness, stochastic partial differential equations

[37L55](#) Infinite-dimensional random dynamical systems; stochastic equations

[60H15](#) Stochastic partial differential equations (aspects of stochastic analysis)

[93B05](#) Controllability

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**Keywords:**

forward compactness; forward controller; pullback attractor

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