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Foliations on unitary Shimura varieties in positive characteristic. (English) Zbl 1457.11086

Summary: When \( p \) is inert in the quadratic imaginary field \( E \) and \( m < n \), unitary Shimura varieties of signature \((n, m)\) and a hyperspecial level subgroup at \( p \), carry a natural foliation of height 1 and rank \( m^2 \) in the tangent bundle of their special fiber \( S \). We study this foliation and show that it acquires singularities at deep Ekedahl-Oort strata, but that these singularities are resolved if we pass to a natural smooth moduli problem \( S^\#(p) \) of a certain Shimura variety with parahoric level structure at \( p \). As a result, we get that this ‘horizontal component’ of \( S^\#(p) \), as well as its multiplicative counterpart, are non-singular (formerly they were only known to be normal and Cohen-Macaulay). We study two kinds of integral manifolds of the foliation: unitary Shimura subvarieties of signature \((m, m)\), and a certain Ekedahl-Oort stratum that we denote \( S^\text{fol} \). We conjecture that these are the only integral submanifolds.

MSC:

11G18 Arithmetic aspects of modular and Shimura varieties
14G35 Modular and Shimura varieties

Keywords: Shimura varieties; Ekedahl-Oort strata; foliations

Full Text: DOI arXiv

References:
