Külshammer, Burkhard
Centers and radicals of group algebras and blocks. (English) Zbl 1457.20004
Arch. Math. 114, No. 6, 619-629 (2020).

Let $B$ be a block of positive defect in the group algebra $FG$ of a finite group $G$ over an algebraically closed field $F$ of prime characteristic $p$. Let $Z(B)$ (resp. $J(B)$) denote the center (resp. the Jacobson radical) of $B$. The author considers the following three related conditions:

$(P_1)$ $J(B) \subseteq Z(B)$;

$(P_2)$ $J(B)$ is commutative; and

$(P_3)$ $J(Z(B))$ is an ideal of $B$.

Let $\overline{G}$ denote the quotient group $G/O_p(G)$.

The author shows that $B$ satisfies $(P_3)$ if and only if $B$ has a one-dimensional module and one of the following conditions holds: (i) $\overline{G}$ is an abelian $p$-group; or (ii) $p = 2$, $\overline{G}$ is a 2-group and $|Z(\overline{G})| = |\overline{G}| = 2$; or (iii) $\overline{G} \cong AGL(1, p^n)$ for some $n \in \mathbb{N}$. The author also shows that if the defect groups of $B$ have order bigger than 2, then $B$ satisfies $(P_2)$ if and only if $B$ has a one-dimensional module and $\overline{G}$ is an abelian $p$-group. Finally, he shows that $B$ satisfies $(P_1)$ if and only if $B$ has a one-dimensional module and $\overline{G}$ is an abelian $p$-group.

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MSC:

20C05 Group rings of finite groups and their modules (group-theoretic aspects)
20C20 Modular representations and characters

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References:
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