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Summary: We study the Nikolskii constant (or the Jackson-Nikolskii constant) for complex trigonometric polynomials in the space $L_p^\alpha(\mathbb{T})$ for $p \geq 1$ with the periodic Gegenbauer weight $|\sin x|^{2\alpha+1}$:

$$C_{p,\alpha}(n) = \sup_{T \in \mathbb{T}\setminus\{0\}} \frac{\|T\|_\infty}{\|T\|_p},$$

where $\|\cdot\|_p = \|\cdot\|_{L_p^\alpha(\mathbb{T})}$. D. Jackson [Bull. Am. Math. Soc. 39, 889–906 (1933; JFM 59.0307.01)] proved that $C_{p,-1/2}(n) \leq c_p n^{1/p}$ for all $n \geq 1$. The problem of finding $C_{p,-1/2}(n)$ has a long history. However, sharp constants are known only for $p = 2$. For $p = 1$, the problem has interesting applications, e.g., in number theory. We note the results of Ja. L. Geronimus, L. V. Taikov, D. V. Gorbachev, I. E. Simonov, P. Yu. Glazyrina. For $p > 0$, we note the results of I. I. Ibragimov, V. I. Ivanov, E. Levin, D. S. Lubinsky, M. I. Ganzburg, S. Yu. Tikhonov, in the weight case – V. V. Arestov, A. G. Babenko, M. V. Deikalova, Á. Horváth.

It is proved that the supremum here is achieved on a real even trigonometric polynomial with a maximum modulus at zero. As a result, a connection is established with the Nikolskii algebraic constant with weight $(1-x^2)^\alpha$, investigated by V. Arestov and M. Deikalova [Comput. Methods Funct. Theory 15, No. 4, 689–708 (2015; Zbl 1329.41012)]. The proof follows their method and is based on the positive generalized translation operator in the space $L_p^\alpha(\mathbb{T})$ with the periodic Gegenbauer weight. This operator was constructed and studied by D. V. Chertova [“Jackson theorems in $L_p$ spaces, $1 \leq p \leq 2$ with periodic Jacobi weight”, Izv. Tul. Gos. Univ. Estestv. Nauki, No. 1, 5–27 (2009)]. As an application, we propose an approach to computing $C_{p,\alpha}(n)$ based on the Arestov-Deikalova duality relations.

MSC:

42A05 Trigonometric polynomials, inequalities, extremal problems
41A17 Inequalities in approximation (Bernstein, Jackson, Nikol’skii-type inequalities)

Keywords:
trigonometric polynomial; algebraic polynomial; Nikolskii constant; Gegenbauer weight

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