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The aim is to construct new solvers for partial differential equations (PDE) based on convolutional neural networks. One considers the classical Poisson equation with Dirichlet boundary condition, \( \Delta u = f \) in \( \Omega \), \( u = 0 \) on \( \partial \Omega \), \( \Omega \) being the two-dimensional unit disk.

The solution can be expressed by means of the Green function \( G(x, y) \), \( u(x) = \int_{\Omega} G(x, y)f(y)dy \), for \( x \in \Omega \).

A discretization procedure is applied and the purpose is to define a neural network which approximates the discretized solution mapping. The third section of the article is devoted to the presentation of the network architecture, the probabilistic training procedure, network loss functions, data generation. Numerical results, comparison of training procedures, uncertainty quantification and inference speed are shown in the fourth section and a brief summary on the developed ConvPDE-UQ framework in the fifth section.

Appendix A presents results from the extension of the method to variable coefficients differential operators or Neumann boundary conditions. Appendix B deals with alternative distributions for modeling network uncertainty.

Reviewer: Claudia Simionescu-Badea (Wien)

MSC:
65N75 Probabilistic methods, particle methods, etc. for boundary value problems involving PDEs
65N80 Fundamental solutions, Green's function methods, etc. for boundary value problems involving PDEs
35J05 Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
68T05 Learning and adaptive systems in artificial intelligence
68T07 Artificial neural networks and deep learning
82C32 Neural nets applied to problems in time-dependent statistical mechanics

Keywords:
partial differential equations; uncertainty quantification; convolutional encoder-decoder networks; deep learning; machine learning; confidence interval

Software:
Unicorn; DGM ; FEniCS; U-Net; SyFi; UFC; Adam; FFC; UFL; DOLFIN; FIAT

Full Text: DOI

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