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A combinatorial classification of 2-regular simple modules for Nakayama algebras. (English)
Zbl 1458.16011

Summary: H. Enomoto [Adv. Math. 335, 838–877 (2018; Zbl 1419.18017)] showed for finite dimensional algebras that the classification of exact structures on the category of finitely generated projective modules can be reduced to the classification of 2-regular simple modules. In this article, we give a combinatorial classification of 2-regular simple modules for Nakayama algebras and we use this classification to answer several natural questions such as when there is a unique exact structure on the category of finitely generated projective modules for Nakayama algebras. We also classify 1-regular simple modules, quasi-hereditary Nakayama algebras and Nakayama algebras of global dimension at most two. It turns out that most classes are enumerated by well-known combinatorial sequences, such as Fibonacci, Riordan and Narayana numbers. We first obtain interpretations in terms of the Auslander-Reiten quiver of the algebra using homological algebra, and then apply suitable bijections to relate these to combinatorial statistics on Dyck paths.

MSC:
16G10 Representations of associative Artinian rings
16E10 Homological dimension in associative algebras
05A19 Combinatorial identities, bijective combinatorics
05A15 Exact enumeration problems, generating functions

Keywords:
Nakayama algebras; quiver representation theory; homological algebra; Dyck paths; bijective combinatorics; combinatorial statistics

Software:
SageMath; QPA ; FindStat; OEIS

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References: