Summary: The determination of exponentially stable equilibria and their basin of attraction for a dynamical system given by a general autonomous ordinary differential equation can be achieved by means of a contraction metric. A contraction metric is a Riemannian metric with respect to which the distance between adjacent solutions decreases as time increases. The Riemannian metric can be expressed by a matrix-valued function on the phase space.

The determination of a contraction metric can be achieved by approximately solving a matrix-valued partial differential equation by mesh-free collocation using Radial Basis Functions (RBF). However, so far no rigorous verification that the computed metric is indeed a contraction metric has been provided.

In this paper, we combine the RBF method to compute a contraction metric with the CPA method to rigorously verify it. In particular, the computed contraction metric is interpolated by a continuous piecewise affine (CPA) metric at the vertices of a fixed triangulation, and by checking finitely many inequalities, we can verify that the interpolation is a contraction metric. Moreover, we show that, using sufficiently dense collocation points and a sufficiently fine triangulation, we always succeed with the construction and verification. We apply the method to two examples.

MSC:

65N35 Spectral, collocation and related methods for boundary value problems involving PDEs
65N15 Error bounds for boundary value problems involving PDEs
65D12 Numerical radial basis function approximation
37B25 Stability of topological dynamical systems
34D20 Stability of solutions to ordinary differential equations

Keywords:
contraction metric; Lyapunov stability; basin of attraction; radial basis functions; reproducing kernel Hilbert spaces; continuous piecewise affine interpolation

Software:
Matlab; SMRSOF; Sostools

Full Text: DOI arXiv

References:


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