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Regularity lemma for distal structures. (English) Zbl 1459.03041

Summary: It is known that families of graphs with a semialgebraic edge relation of bounded complexity satisfy much stronger regularity properties than arbitrary graphs, and can be decomposed into very homogeneous semialgebraic pieces up to a small error (see e.g. [J. Pach and J. Solymosi, J. Comb. Theory, Ser. A 96, No. 2, 316–325 (2001; Zbl 0989.05031); N. Alon et al., J. Comb. Theory, Ser. A 111, No. 2, 310–326 (2005; Zbl 1099.14048); J. Fox et al., J. Reine Angew. Math. 671, 49–83 (2012; Zbl 1306.05171); J. Fox et al., SIAM J. Comput. 45, No. 6, 2199–2223 (2016; Zbl 1353.05090)]). We show that similar results can be obtained for families of graphs with the edge relation uniformly definable in a structure satisfying a certain model-theoretic property called distality, with respect to a large class of generically stable measures. Moreover, distality characterizes these strong regularity properties. This applies in particular to graphs definable in arbitrary o-minimal structures and in p-adics.

MSC:
03C45 Classification theory, stability, and related concepts in model theory
03C65 Models of other mathematical theories
05C65 Hypergraphs
05D10 Ramsey theory
05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
14P10 Semialgebraic sets and related spaces
03C64 Model theory of ordered structures; o-minimality

Keywords:
NIP; VC-dimension; distal theories; o-minimality; p-adics; Erdős-Hajnal conjecture; regularity lemma

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