Open problems in wavelet theory. (English) Zbl 1459.42050

Part of the abstract is as follows: “We present a collection of easily stated open problems in wavelet theory and we survey the current status of answering them.”

This is a clear general description of the contents. The paper is really interesting, well written and, reading it, one can learn many things. The authors gathered several problems, partially or completely solved, or still unsolved.

Section 1 is the introduction. Section 2 is concerned by one dimensional wavelets.

The first subsection deals with connectivity:

(*) Is the collection of all orthonormal wavelets (as a subset of the unit sphere in $L^2(\mathbb{R})$) path connected in $L^2(\mathbb{R})$ norm?

This problem remains open in full generality but significant progress have been obtained, such as a positive answer if the wavelet comes from an MRA (multi-resolution analysis). A positive result for MSF wavelet (minimally supported frequency wavelet) has also been obtained. Variants of the question (*) have also been studied for other classes of wavelets.

Then the other subsections deal with wavelets for the Hardy space $H^2(\mathbb{R})$, with minimality of MSF wavelets, density of Riesz wavelets, intersection of negative dilates, and extension of wavelet frames.

Higher dimensional wavelets are studied in section 3. Here are lines of the introducing paragraph: “Most of the one dimensional problems discussed in Section 2 have higher dimensional analogues. Rather surprisingly, their higher dimensional analogues have definitive answers for certain classes of dilation matrices. Subsequently, we shall focus on problems which have been resolved in one or two dimensions, but remain open in higher dimensions.”

Section 4 consists in a proof of a theorem stated in section 2 and which concerns wavelet sets, closely related with MSF wavelet.

The list of references is quite large, which is natural for such an expository paper.

For the entire collection see [Zbl 1455.47001].

Reviewer: Françoise Bastin (Liège)

MSC:

42C40 Nontrigonometric harmonic analysis involving wavelets and other special systems

46C05 Hilbert and pre-Hilbert spaces: geometry and topology (including spaces with semidefinite inner product)

Keywords:

minimally supported frequency wavelets; MRA wavelets

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