

Bakker, Benjamin; Lehn, Christian

A global Torelli theorem for singular symplectic varieties. (English) Zbl 1460.32014
J. Eur. Math. Soc. (JEMS) 23, No. 3, 949-994 (2021).

Summary: We systematically study the moduli theory of symplectic varieties (in the sense of Beauville) which admit a resolution by an irreducible symplectic manifold. In particular, we prove an analog of Verbitsky's global Torelli theorem for the locally trivial deformations of such varieties. Verbitsky's work on ergodic complex structures replaces twistor lines as the essential global input. In so doing we extend many of the local deformation-theoretic results known in the smooth case to such (not-necessarily-projective) symplectic varieties. We deduce a number of applications to the birational geometry of symplectic manifolds, including some results on the classification of birational contractions of $K3^{[n]}$ -type varieties.

MSC:

- [32G13](#) Complex-analytic moduli problems
- [53C26](#) Hyper-Kähler and quaternionic Kähler geometry, "special" geometry
- [14B07](#) Deformations of singularities
- [14J10](#) Families, moduli, classification: algebraic theory
- [32S45](#) Modifications; resolution of singularities (complex-analytic aspects)
- [32S15](#) Equisingularity (topological and analytic)

Cited in **1** Review
Cited in **9** Documents

Keywords:

Hyperkähler manifold; symplectic variety; global Torelli theorem; locally trivial deformation

Full Text: [DOI](#) [arXiv](#)

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