Sufficiently collapsed irreducible Alexandrov 3-spaces are geometric.

The main result of the paper generalizes the result by T. Shioya and T. Yamaguchi on collapsing closed 3-dimensional Riemannian manifolds with curvatures bounded below, Corollary 0.9 in [J. Differ. Geom. 56, No. 1, 1–66 (2000; Zbl 1036.53028)]. Similar to the case of Riemannian manifolds, a closed Alexandrov 3-space is called irreducible if every embedded 2-sphere in it bounds a 3-ball; in addition if the space has topologically singular points, it is required that every 2-sided $RP^2$ bound a $K(RP^2)$, a cone over $RP^2$.

The main result states that given $D > 0$, there is a positive $\varepsilon(D)$ such that, if $M$ is a closed irreducible Alexandrov 3-space of curvature $\geq -1$, $\text{diam}(M) \leq D$ and $\text{Vol}(M) < \varepsilon(D)$, then $M$ admits a geometric structure modeled on one of the seven geometries $\mathbb{R}^3$, $S^3$, $S^2 \times \mathbb{R}$, $H^2 \times \mathbb{R}$, $\tilde{SL}_2(\mathbb{R})$, Nil and Sol.

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