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**Bass numbers of local cohomology of cover ideals of graphs.** (English) Zbl 1461.13017  
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The present paper studies Bass numbers (and in particular Lyubeznik numbers) of local cohomology modules supported on cover ideals of simple graphs. The authors use splitting techniques to relate the Lyubeznik numbers of a graph to the Lyubeznik numbers of the subgraph obtained by removing a vertex. This work allows to compute all the Bass numbers of local cohomology modules considering subgraphs of the initial graph. In particular this allows to describe the structure of the injective resolution of these modules in terms of the number of connected components of the corresponding subgraphs. Finally, the authors deduce vanishing criteria for local cohomology modules depending of those connected components.

Reviewer: [Eduardo Saenz-de-Cabezón \(Logroño\)](#)

**MSC:**

- 13D45 Local cohomology and commutative rings
- 13C11 Injective and flat modules and ideals in commutative rings
- 05C38 Paths and cycles
- 05C40 Connectivity

**Keywords:**

[local cohomology](#); [injective resolution](#); [graphs](#); [connectivity](#)

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