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On a property of harmonic measure on simply connected domains. (English) Zbl 1461.30058

Summary: Let $D \subset \mathbb{C}$ be a domain with $0 \in D$. For $R > 0$, let $\tilde{\omega}_D(R)$ denote the harmonic measure of $D \cap \{|z| = R\}$ at 0 with respect to the domain $D \cap \{|z| < R\}$ and let $\omega_D(R)$ denote the harmonic measure of $\partial D \cap \{|z| \geq R\}$ at 0 with respect to $D$. The behavior of the functions $\omega_D$ and $\tilde{\omega}_D$ near $\infty$ determines (in some sense) how large $D$ is. However, it is not known whether the functions $\omega_D$ and $\tilde{\omega}_D$ always have the same behavior when $R$ tends to $\infty$. Obviously, $\omega_D(R) \leq \tilde{\omega}_D(R)$ for every $R > 0$. Thus, the arising question, first posed by D. Betsakos [Rocky Mt. J. Math. 31, No. 3, 773–795 (2001; Zbl 1006.31002)], is the following: Does there exist a positive constant $C$ such that for all simply connected domains $D$ with $0 \in D$ and all $R > 0$,

$$\omega_D(R) \geq C\tilde{\omega}_D(R)?$$

In general, we prove that the answer is negative by means of two different counter-examples. However, under additional assumptions involving the geometry of $D$, we prove that the answer is positive. We also find the value of the optimal constant for starlike domains.

MSC:
30C85 Capacity and harmonic measure in the complex plane
30F45 Conformal metrics (hyperbolic, Poincaré, distance functions)
30C35 General theory of conformal mappings
31A15 Potentials and capacity, harmonic measure, extremal length and related notions in two dimensions

Keywords:
harmonic measure; conformal mapping; hyperbolic distance

Full Text: DOI arXiv

References:


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