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Complete semialgebraic invariant synthesis for the Kannan-Lipton orbit problem. (English)

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Summary: The Orbit Problem consists of determining, given a matrix $A$ on $\mathbb{Q}^d$, together with vectors $x$ and $y$, whether the orbit of $x$ under repeated applications of $A$ can ever reach $y$. This problem was famously shown to be decidable by R. Kannan and R. J. Lipton’s Orbit Problem [in: Proceedings of the 12th annual ACM symposium on theory of computing, STOC ’80. New York, NY: Association for Computing Machinery (ACM). 252–261 (1980; doi:10.1145/800141.804673); J. Assoc. Comput. Mach. 33, 808–821 (1986; Zbl 1326.68162)] in the 1980s. In this paper, we are concerned with the problem of synthesising suitable invariants $P \subseteq \mathbb{R}^d$, i.e., sets that are stable under $A$ and contain $x$ but not $y$, thereby providing compact and versatile certificates of non-reachability. We show that whether a given instance of the Orbit Problem admits a semialgebraic invariant is decidable, and moreover in positive instances we provide an algorithm to synthesise suitable succinct invariants of polynomial size. Our results imply that the class of closed semialgebraic invariants is closure-complete: there exists a closed semialgebraic invariant if and only if $y$ is not in the topological closure of the orbit of $x$ under $A$.

MSC:

37C35 Orbit growth in dynamical systems
37C79 Symmetries and invariants of dynamical systems
68U05 Computer graphics; computational geometry (digital and algorithmic aspects)

Keywords:
verification; algebraic computation; Skolem problem; orbit problem; invariants

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References:


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