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Forward mortality rates in discrete time. I: Calibration and securities pricing. (English)

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Summary: Many users of mortality models are interested in using them to place values on longevity-linked liabilities and securities. Modern regulatory regimes require that the values of liabilities and reserves are consistent with market prices (if available), though the gradual emergence of a traded market in longevity risk needs methods for pricing new types of longevity-linked securities quickly and efficiently. In this study, we develop a new forward mortality framework to enable the efficient pricing of longevity-linked liabilities and securities in a market-consistent fashion. This approach starts from the historical data of the observed mortality rates, i.e., the force of mortality. Building on the dynamics of age/period/cohort models of the observed force of mortality, we develop models of forward mortality rates and then use a change of measure to incorporate whatever market information is available. The resulting forward mortality rates are then used to value a number of different longevity-linked securities, such as q-forwards, s-forwards, and longevity swaps.

For Part II, see [the authors, ibid. 25, Suppl. 1, S508–S533 (2021; Zbl 1461.91247)].

MSC:

91G05 Actuarial mathematics
91G20 Derivative securities (option pricing, hedging, etc.)

Full Text: DOI

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