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Certain monomial ideals whose numbers of generators of powers descend. (English)
Zbl 1462.13022

Summary: This paper studies the numbers of minimal generators of powers of monomial ideals in polynomial rings. For a monomial ideal $I$ in two variables, S. Eliahou et al. [J. Algebra 514, 99–112 (2018; Zbl 1403.13033)] gave a sharp lower bound $\mu(I^2) \geq 9$ for the number of minimal generators of $I^2$ with $\mu(I) \geq 6$. Recently, O. Gasanova [Commun. Algebra 48, No. 11, 4824–4831 (2020; Zbl 1441.13005)] constructed monomial ideals such that $\mu(I) > \mu(I^n)$ for any positive integer $n$. In reference to them, we construct a certain class of monomial ideals such that $\mu(I) > \mu(I^2) > \cdots > \mu(I^n) = (n+1)^2$ for any positive integer $n$, which provides one of the most unexpected behaviors of the function $\mu(I^k)$. The monomial ideals also give a peculiar example such that the Cohen-Macaulay type (or the index of irreducibility) of $R/I^n$ descends.

MSC:
13F55 Commutative rings defined by monomial ideals; Stanley-Reisner face rings; simplicial complexes
13D40 Hilbert-Samuel and Hilbert-Kunz functions; Poincaré series
13F20 Polynomial rings and ideals; rings of integer-valued polynomials

Keywords:
number of generators; polynomial ring; monomial ideal

Software:
CoCoA

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References:

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