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Rational points on log Fano threefolds over a finite field. (English) Zbl 1462.14045

J. Eur. Math. Soc. (JEMS) 21, No. 12, 3759-3795 (2019).

Summary: We prove the $W\mathcal{O}$ -rationality of klt threefolds and the rational chain connectedness of klt Fano threefolds over a perfect field of characteristic $p > 5$. As a consequence, any klt Fano threefold over a finite field has a rational point.

MSC:

[14J45](#) Fano varieties
[14G05](#) Rational points
[14E30](#) Minimal model program (Mori theory, extremal rays)

Cited in **1** Review
Cited in **12** Documents

Keywords:

[rational points](#); [Fano varieties](#); [Witt rationality](#); [rational chain connectedness](#)

Full Text: [DOI](#) [arXiv](#)

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