dos Santos, E. B.; Leitão, R.

On the Hölder regularity for solutions of integro-differential equations like the anisotropic fractional Laplacian. (English) Zbl 1462.35115

SN Partial Differ. Equ. Appl. 2, No. 2, Paper No. 25, 34 p. (2021).

Summary: In this paper we study integro-differential equations like the anisotropic fractional Laplacian. As in [*L. Silvestre*, Indiana Univ. Math. J. 55, No. 3, 1155–1174 (2006; Zbl 1101.45004)], we adapt the De Giorgi technique to achieve the C^γ-regularity for solutions of class C^2 and use the geometry found in [*L. A. Caffarelli* et al., Math. Ann. 360, No. 3–4, 681–714 (2014; Zbl 1304.35730)] to get an ABP estimate, a Harnack inequality and the interior $C^{1,\gamma}$ regularity for viscosity solutions.

MSC:

35B65 Smoothness and regularity of solutions to PDEs
35D40 Viscosity solutions to PDEs
35R09 Integro-partial differential equations
35R11 Fractional partial differential equations
35J61 Semilinear elliptic equations
35J70 Degenerate elliptic equations

Keywords:

ABP estimate; Harnack inequality; interior $C^{1,\gamma}$ regularity

Full Text: DOI

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