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A coefficient determination in nonlocal problem for Boussinesq type integro-differential equation with degenerate kernel. (Russian. English summary) Zbl 1463.35011
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Summary: In the three-dimensional domain a Boussinesq type linear integro-differential equation of the fourth order with a restore coefficient and a degenerate kernel is considered. The solution of this integro-differential equation is considered in the class of continuously differentiable functions. First, we study the classical solvability of a nonlocal direct boundary value problem for the considered Boussinesq integro-differential equation with a parameter in the integral term. The method of separation of variables and the method of a degenerate kernels are used. A countable system of algebraic equations is obtained. The solution of this algebraic system of equations for regular values of the spectral parameter in the integral term of a given equation allows us to construct a solution of a non-local direct boundary value problem for an integro-differential equation in the form of a Fourier series. A criterion for the unique solvability of a direct boundary value problem is established for fixed values of the restore function. Using the Cauchy-Bunyakovsky inequality and the Bessel inequality, we prove the absolute and uniform convergence of the obtained Fourier series. The continuity of all the derivatives of the solution of the direct boundary value problem for a given equation is also proved. Further, with the help of an additional integral condition, the restore function is uniquely determined in the form of a Fourier series. The criterion of continuity of second order derivatives of the restore function with respect to space variables is established. Based on the found values of the restore function, the main unknown function is uniquely determined as a solution to the inverse problem for the considering integro-differential equation. In addition, the stability with respect to restore function of the solution of an integro-differential equation is studied.

MSC:

35A02	Uniqueness problems for PDEs: global uniqueness, local uniqueness, non-uniqueness	Cited in **6** Documents
35M10	PDEs of mixed type	
35S05	Pseudodifferential operators as generalizations of partial differential operators	

Keywords:

Boussinesq type integro-differential equation; fourth-order equation; degenerate kernel; integral conditions; one valued solvability

Full Text: DOI MNR

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