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On isomorphism of some functional spaces under action of integro-differential operators. (Russian. English summary) Zbl 1463.35164

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Summary: In the paper we consider representations of the second kind for solutions to the linear general uniform first order elliptic system in the unit circle $D = \{z : |z| \leq 1\}$ written in terms of complex functions:

$$\mathcal{D}w \equiv \partial_{\bar z}w + q_1(z)\partial_z w + q_2(z)\partial_{\bar z}\overline{w} + A(z)w + B(z)\overline{w} = R(z),$$

where $w = w(z) = u(z) + iv(z)$ is the sought complex function, $q_1(z)$ and $q_2(z)$ are given measurable complex functions satisfying the uniform ellipticity condition of the system:

$$|q_1(z)| + |q_2(z)| \leq q_0 = \text{const} < 1, \quad z \in \overline{D},$$

and $A(z), B(z), R(z) \in L_p(\overline{D})$, $p > 2$, are also given complex functions.

The representation of the second kind is based on the well-known Pompeiu's formula: if $w \in W_p^1(\overline{D})$, $p > 2$, then

$$w(z) = \frac{1}{2\pi i}\int\limits_{\Gamma} \frac{w(\zeta)}{\zeta - z}d\zeta - \frac{1}{\pi}\iint\limits_{D} \frac{\partial w}{\partial \bar z} \cdot \frac{d\xi d\eta}{\zeta - z},$$

where $w(z) \in W_p^1(\overline{D})$, $p > 2$. Then for the solution $w(z)$ we can write the representation

$$\Omega(w) = \frac{1}{2\pi i}\int\limits_{\Gamma} \frac{w(\zeta)}{\zeta - z}d\zeta + TR(z)$$

where

$$\Omega(w) \equiv w(z) + T(q_1(z)\partial_z w + q_2(z)\partial_{\bar z}\overline{w} + A(z)w + B(z)\overline{w}).$$

Under appropriate assumptions about on coefficients we prove that Ω is the isomorphism of the spaces $C_\alpha^k(\overline{D})$ and $W_p^k(\overline{D})$, $k \geq 1$, $0 < \alpha < 1$, $p > 2$. These results develop and complete B. V. Boyarsky's works, where representations of the first kind were obtained. Also this work complete author's results on representations of the second kind with more difficult operators. As an implication of the properties of the operator Ω, we obtain apriori estimates for the norms $\|w\|_{C_\alpha^{k+1}(\overline{D})}$ and $\|w\|_{W_p^k(\overline{D})}$.

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35C15 Integral representations of solutions to PDEs

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References:

[1] I.N. Vekua, Generalized analytic functions, Int. Ser. Monog. Pure Appl. Math., 25, Pergamon Press, Oxford, 1962 · Zbl 0100.07603

[2] B.V. Boyarsky, "Generalized solutions of a system of differential equations of first order and of elliptic type with discontinuous coefficients", Matem. Sborn., 43:4 (1957), 451-503 (in Russian)

[3] S.B. Klimentov, "On a method of constructing the solutions of boundary-value problems of the theory of bendings of surfaces of positive curvature", Journal of Mathematical Sciences, 51:2 (1990), 2230-2248 · Zbl 0716.35020

[4] V.S. Vinogradov, "On the solvability of a singular integral equation", Sov. Math. Dokl., 19 (1978), 827-829 · Zbl 0423.45001

[5] V.S. Vinogradov, “Structure of regularizers for elliptic boundary-value problems in the plane”, Differ. Equat., 26:1 (1990), 14-20 · Zbl 0708.35028

[6] S.B. Klimentov, “Another version of Kellogg”s theorem”, Complex Variables and Elliptic Equations, 60:12 (2015), 1647-1657 · Zbl 1332.30017 · doi:10.1080/17476933.2015.1038529

[7] G.S. Litvinchuk, Solvability theory of boundary value problems and singular integral equations with shift, Math. Appl., 523, Kluwer Academic Publ., Dordrecht, 2000 · Zbl 0980.45001

[8] S.B. Klimentov, “On combinations of the circle shifts and some one-dimensional integral operators”, Vladikavkaz. Matem. Zhurn., 19:1 (2017), 30-40 (in Russian) · Zbl 07259887

[9] S. Agmon, A. Douglis, L. Nirenberg, “Estimates near the boundary for solutions of elliptic partial differential equations satisfying general boundary conditions”, Comm. Pure Appl. Math., XII (1959), 623-727 · Zbl 0093.10401 · doi:10.1002/cpa.3160120405

[10] V.N. Monakhov, Boundary-value problems with free boundaries for elliptic systems of equations, Transl. Math. Monog., 57, Amer. Math. Soc., Providence, R.I., 1983 · Zbl 0532.35001 · doi:10.1090/mmono/057

[11] V.A. Danilov, “Estimates of distortion of quasiconformal mapping in space \(C^m_\alpha \)”, Siber. Math. J., 14:3 (1973), 362-369 · Zbl 0267.30019 · doi:10.1007/BF00967612

[12] S. Sternberg, Lectures on differential geometry, Prentice-Hall Inc., Englewood Cliffs, 1964 · Zbl 0129.13102

[13] F.D. Gakhov, Boundary value problems, Dover Publications, New York, 1990 · Zbl 0830.30026

[14] S.B. Klimentov, “Representations of the “second kind” for the Hardy classes of solutions to the Beltrami equation”, Siber. Math. J., 55:2 (2014), 262-275 · Zbl 1302.30061 · doi:10.1134/S0037446614020098

[15] S.B. Klimentov, “The Riemann-Hilbert problem in Hardy classes for general first-order elliptic systems”, Russ. Math. Izv. VUZ, 60:6 (2016), 29-39 · Zbl 1345.30044 · doi:10.3103/S1066369X16060049

[16] M.A. Naimark, Normed rings, Wolters-Noordhoff Publishing, Groningen, 1968 · Zbl 0073.08902