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Products of eigenfunctions and Wronskians. (Russian. English summary) Zbl 1463.35384
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Summary: We consider new Wronskian identities found recently in Maikop city. We discuss the relations of these identities with the theory of integrable systems and with the general theory of invertible Darboux transforms for linear differential operators with one independent variable. The object of our study are relations of two Wronskians of orders N and $N' > N$ homogeneous with respect to a dilatation group. The elements of the first Wronskian of the order N are arbitrary functions, which enlarges essentially the means of the theory, while the elements of other Wropnskian are formed by the products of these functions of a given order $n \geqslant 2$.

The dilatation group allows us to pass to projective coordinates in the considered quotient of the Wronskians and in particular, to include symmetric functions and polynomials into the considered theory.

The simplest case is naturally $N = 2$, in which the second Wronskian turns out to be a power of the original Wronskian and hence, the considered quotient becomes independent of the choice of the elements in the second Wronskian. In this case we obtain new equations for the third and other powers of the eigenfunctions of an one-dimensional Schrödinger equation generalizing known formulae for squares related with the Schwarz derivative and KdV hierarchy.

The case $N = 3$ seems to be very interesting from various points of view, but to study it, a further developing of the methods in the projective theory of Wronskians is needed by employing logarithmic derivatives and their higher analogues.

MSC:

35P05 General topics in linear spectral theory for PDEs
35B10 Periodic solutions to PDEs

Keywords:

factorization; Wronski matrix; Schwarz derivative; Riccati equation; Darboux transform

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