Hong, Qingguo; **Kraus, Johannes**; **Lymbery, Maria**; **Philo, Fadi**

Conservative discretizations and parameter-robust preconditioners for Biot and multiple-network flux-based poroelasticity models. (English) Zbl 1463.65372

Numer. Linear Algebra Appl. 26, No. 4, e2242, 25 p. (2019).

Summary: The parameters in the governing system of partial differential equations of multiple-network poroelasticity models typically vary over several orders of magnitude, making its stable discretization and efficient solution a challenging task. In this paper, we prove the uniform Ladyzhenskaya-Babuška-Brezzi (LBB) condition and design uniformly stable discretizations and parameter-robust preconditioners for flux-based formulations of multiporosity/multipermeability systems. Novel parameter-matrix-dependent norms that provide the key for establishing uniform LBB stability of the continuous problem are introduced. As a result, the stability estimates presented here are uniform not only with respect to the Lamé parameter λ but also to all the other model parameters, such as the permeability coefficients K_i; storage coefficients c_{p_i}; network transfer coefficients β_{ij}, $i, j = 1, \ldots, n$; the scale of the networks n; and the time step size τ. Moreover, strongly mass-conservative discretizations that meet the required conditions for parameter-robust LBB stability are suggested and corresponding optimal error estimates proved. The transfer of the canonical (norm-equivalent) operator preconditioners from the continuous to the discrete level lays the foundation for optimal and fully robust iterative solution methods. The theoretical results are confirmed in numerical experiments that are motivated by practical applications.

MSC:

65N30	Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs	Cited in **21** Documents
65N12	Stability and convergence of numerical methods for boundary value problems involving PDEs	
65F08	Preconditioners for iterative methods	
65F10	Iterative numerical methods for linear systems	
76S05	Flows in porous media; filtration; seepage	
74F10	Fluid-solid interactions (including aero- and hydro-elasticity, porosity, etc.)	

Keywords:

Biot's consolidation model; multiple-network poroelastic theory (MPET); parameter-robust LBB stability; robust norm-equivalent preconditioners; strongly mass-conservative discretization

Software:

FEniCS

Full Text: DOI arXiv