Let $G$ be a finite non-cyclic group that can be generated by two elements. In [Mich. Math. J. 22, 53–64 (1975; Zbl 0294.20035)], J. L. Brenner and J. Wiegold defined the spread of $G$, denoted $s(G)$, to be the largest integer $k$ such that for any nontrivial elements $x_1, \ldots, x_k$ in $G$, there exists $y \in G$ such that $G = \langle x_i, y \rangle$ for all $i$. This leads naturally to the more restrictive notion of uniform spread, denoted $u(G)$: this is the largest integer $k$ such that there is a conjugacy class $C$ of $G$ with the property that for any nontrivial elements $x_1, \ldots, x_k$ in $G$, there exists $y \in C$ such that $G = \langle x_i, y \rangle$ for all $i$. Clearly, $s(G) \geq u(G)$ and T. Breuer et al. [J. Algebra 320, No. 2, 443–494 (2008; Zbl 1181.20013)] proved that $u(G) \geq 2$ for every non-abelian finite simple group $G$. For any group with $u(G) \geq 1$, the authors define the uniform domination number $\gamma_u(G)$ of $G$ to be the minimal size of a subset $S$ of conjugate elements such that for each nontrivial $x \in G$ there exists $y \in S$ with $G = \langle x, y \rangle$. They establish several new interesting results on the spread, uniform spread and uniform domination number of finite groups and finite simple groups. In particular, they make substantial progress towards a classification of the simple groups $G$ with $\gamma_u(G) = 2$, and they study the associated probability that two randomly chosen conjugate elements form a uniform dominating set for $G$. They also establish new results concerning the 2-generation of soluble groups (in this case $s(G), u(G)$ and $\gamma_u(G)$ are completely determined) and symmetric groups (for example they proved that $u(S_n) = 2$ if $n \neq 6$ and $\gamma_u(S_n) \geq \log_2(n)$ for all $n$). Several interesting open problems are proposed.

Reviewer: Andrea Lucchini (Padova)

MSC:
20F05 Generators, relations, and presentations of groups
20D05 Finite simple groups and their classification
20D06 Simple groups: alternating groups and groups of Lie type
20J05 Cohomology of groups
20J06 Homological methods in group theory

Keywords:
soluble groups; simple groups; symmetric groups; generators; spread; domination number

Software:
GAP; CTblLib; Magma; GAP Character Table Library

Full Text: DOI

References: