This article compares different $C^*$-algebras that are associated to a compactly aligned product system over an Abelian lattice order, equipped with an action by another locally compact group. In this situation, one may first form a Cuntz-Pimsner algebra of the product system; this inherits a group action, so that one may form a crossed product. Or one may first form a crossed product by the group action; this produces a new product system over the same lattice order, which has a Cuntz-Pimsner algebra. It is shown that the resulting $C^*$-algebras are canonically isomorphic (see Theorem 3.8). The proof of the theorem uses non-self-adjoint operator algebras and their $C^*$-envelopes, and thus also proves results about these $C^*$-envelopes along the way.

The Cuntz-Pimsner algebra of a product system over an Abelian lattice order $(G, P)$ carries a gauge action of the dual group $\hat{G}$. The crossed product for this dual gauge action is identified with the Cuntz-Pimsner algebra of another product system over $(G, P)$. This is analogous to the classical Takai Duality Theorem. A key step to prove the latter result is to extend a suitable product system of equivalence bimodules over $P$ to a Fell bundle over the ambient group $G$. This construction is studied in greater generality in the dissertation of C. Sehnem [On $C^*$-algebras associated to product systems. (2018), http://hdl.handle.net/11858/00-1735-0000-002E-E3EC-A]. Several results in this article seem closely related to results of Sehnem.

Along the way, the article characterises compactly aligned product systems by a more checkable condition (see Proposition 3.2). This implies that taking the crossed product by a suitable group action preserves compact alignment of product systems (see Proposition 3.6).

Reviewer: Ralf Meyer (Göttingen)

MSC:

46L55 Noncommutative dynamical systems

Keywords:

Cuntz-Pimsner algebra; Nica-Toeplitz algebra; compactly aligned; product system; Takai duality

Full Text: DOI

References:


This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.