Let $M$ be a closed manifold, $\dim M \geq 2$, and $G$ be a finite digraph with a so-called good orientation. The author considers the following realization problem: Is $G$ homeomorphic to the Reeb graph $R_f$ of some Morse function $f$ on $M$? He proves that this is true if and only if 

\[ b_1(G) \leq R(M), \]

where $b_1(G)$ is the cycle rank of the graph and $R(M)$ is the maximum cycle rank among all Reeb graphs of smooth functions on $M$ with finite number of critical points. Moreover, any integer $r \in [0, R(M)]$ can be realized as the cycle rank of the Reeb graph of a Morse function on $M$; in particular, the above inequality is exact. In addition, $R(M) = \text{corank}(\pi_1(M))$, the corank of the fundamental group of the manifold.

For surfaces, $\dim M = 2$, the author has proved a similar result even up to isomorphism in his previous work [Ł. P. Michalak, Topol. Methods Nonlinear Anal. 52, No. 2, 749–762 (2018; Zbl 1425.58022)].

Reviewer: Irina Gelbukh (Ciudad de México)

MSC:

57M15 Relations of low-dimensional topology with graph theory
05C76 Graph operations (line graphs, products, etc.)
05C38 Paths and cycles
68U10 Computing methodologies for image processing
57R70 Critical points and critical submanifolds in differential topology

Keywords:
Reeb graph; Morse function; corank of the fundamental group

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References:


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