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Doubly periodic lozenge tilings of a hexagon and matrix valued orthogonal polynomials.
(English) [Zbl 1466.05027]


Summary: We analyze a random lozenge tiling model of a large regular hexagon, whose underlying weight structure is periodic of period 2 in both the horizontal and vertical directions. This is a determinantal point process whose correlation kernel is expressed in terms of non-Hermitian matrix valued orthogonal polynomials (OPs). This model belongs to a class of models for which the existing techniques for studying asymptotics cannot be applied. The novel part of our method consists of establishing a connection between matrix valued and scalar valued OPs. This allows to simplify the double contour formula for the kernel obtained by M. Duits and A. B. J. Kuijlaars [J. Eur. Math. Soc. (JEMS) 23, No. 4, 1075–1131 (2021; Zbl 07357345)] by reducing the size of a Riemann-Hilbert problem. The proof relies on the fact that the matrix valued weight possesses eigenvalues that live on an underlying Riemann surface $\mathcal{M}$ of genus 0.

We consider this connection of independent interest; it is natural to expect that similar ideas can be used for other matrix valued OPs, as long as the corresponding Riemann surface $\mathcal{M}$ is of genus 0. The rest of the method consists of two parts, and mainly follows the lines of a previous work of C. Charlier et al. [Commun. Math. Phys. 378, No. 1, 401–466 (2020; Zbl 1446.52017)]. First, we perform a Deift-Zhou steepest descent analysis to obtain asymptotics for the scalar valued OPs. The main difficulty is the study of an equilibrium problem in the complex plane. Second, the asymptotics for the OPs are substituted in the double contour integral and the latter is analyzed using the saddle point method. Our main results are the limiting densities of the lozenges in the disordered flower-shaped region. However, we stress that the method allows in principle to rigorously compute other meaningful probabilistic quantities in the model.

MSC:
05B45 Combinatorial aspects of tesselation and tiling problems
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
33C47 Other special orthogonal polynomials and functions
52C20 Tilings in 2 dimensions (aspects of discrete geometry)

Keywords:
doubly periodic lozenge tilings; matrix valued orthogonal polynomials; Riemann-Hilbert problems

Full Text: DOI