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On minimal Ramsey graphs and Ramsey equivalence in multiple colours. (English) Zbl 1466.05209

Summary: For an integer \( q \geq 2 \), a graph \( G \) is called \( q \)-Ramsey for a graph \( H \) if every \( q \)-colouring of the edges of \( G \) contains a monochromatic copy of \( H \). If \( G \) is \( q \)-Ramsey for \( H \) yet no proper subgraph of \( G \) has this property, then \( G \) is called \( q \)-Ramsey-minimal for \( H \). Generalizing a statement by S. A. Burr et al. [Discrete Math. 54, 1–13 (1985; Zbl 0564.05040)], we prove that, for \( q \geq 3 \), if \( G \) is a graph that is not \( q \)-Ramsey for some graph \( H \), then \( G \) is contained as an induced subgraph in an infinite number of \( q \)-Ramsey-minimal graphs for \( H \) as long as \( H \) is 3-connected or isomorphic to the triangle. For such \( H \), the following are some consequences.

For \( 2 \leq r < q \), every \( r \)-Ramsey-minimal graph for \( H \) is contained as an induced subgraph in an infinite number of \( q \)-Ramsey-minimal graphs for \( H \).

For every \( q \geq 3 \), there are \( q \)-Ramsey-minimal graphs for \( H \) of arbitrarily large maximum degree, genus and chromatic number.

The collection \( \{ M_q(H) : H \text{ is 3-connected or } K_3 \} \) forms an antichain with respect to the subset relation, where \( M_q(H) \) denotes the set of all graphs that are \( q \)-Ramsey-minimal for \( H \).

We also address the question of which pairs of graphs satisfy \( M_q(H_1) = M_q(H_2) \), in which case \( H_1 \) and \( H_2 \) are called \( q \)-equivalent. We show that two graphs \( H_1 \) and \( H_2 \) are \( q \)-equivalent for even \( q \) if they are 2-equivalent, and that in general \( q \)-equivalence for some \( q \geq 3 \) does not necessarily imply 2-equivalence. Finally we indicate that for connected graphs this implication may hold: results by J. Nešetřil and V. Rödl [J. Comb. Theory, Ser. B 20, 243–249 (1976; Zbl 0329.05115)] and by J. Fox et al. [ibid. 120, 64–82 (2016; Zbl 1337.05076)] imply that the complete graph is not 2-equivalent to any other connected graph. We prove that this is the case for an arbitrary number of colours.

MSC:

05D10 Ramsey theory
05C55 Generalized Ramsey theory
05C15 Coloring of graphs and hypergraphs

Keywords:
\( q \)-Ramsey graph; chromatic number

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