Let $G$ be a simple algebraic group over an algebraically closed field. Let $P$ be a parabolic subgroup of $G$ with abelian unipotent radical $P_u$, and let $L$ be a Levi subgroup of $P$. Then $G/L$ is a Hermitian symmetric space. Let $B$ be a Borel subgroup of $G$ contained in $P$. The Bruhat order on Hermitian symmetric varieties referred to in the title of the article refers to the partial order defined by inclusions of $B$-orbit closures in $G/L$. It is named in analogy with the classical Bruhat order, defined by inclusions of Schubert varieties in $G/B$. Such Schubert varieties are parametrized by elements of the Weyl group of $G$ thanks to the Bruhat decomposition, and the induced partial order on the Weyl group can be encoded in a fully combinatorial way.

The $B$-orbits in $G/L$ have already been studied, notably by R. W. Richardson and T. A. Springer [Geom. Dedicata 35, No. 1-3, 389-436 (1990; Zbl 0704.20039)]. They are parametrized by combinatorial data as well. The main result of the paper is a combinatorial translation of the partial order alluded to above. This provides a solution to a conjecture of R. W. Richardson and T. A. Springer [Contemp. Math. 153, 109–142 (1993; Zbl 0840.20039), Conjecture 5.6.2].

One ingredient of the proof, which leads to results of independent interest is the following. The inclusion $L \subset P$ provides a homogeneous fibration of $G/L$ onto $G/P$ whose fibers are isomorphic to $P/L$. Under natural identifications and exponential map, this fiber is isomorphic to the Lie algebra $p_u$ of $P_u$. Orbits under $B$ behave well under this projection, allowing to approach the problem by studying $B$-orbit closures in $G/P$ (a well-known variation on the classical Bruhat order) and $B$-orbit closures in the fibers $p_u$. The latter is studied as a first step in the present article, providing in particular the final steps to settle a conjecture of D. Panyushev [Transform. Groups 22, No. 2, 503–524 (2017; Zbl 1377.22016), Conjecture 6.2].

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- 14M15 Grassmannians, Schubert varieties, flag manifolds

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References:


